TIME-VARYING RELIABILITY PREDICTION OF COMPLEX STRUCTURES USING PDEM with EXTREME LEARNING MACHINE

Sajad Saraygord Afshari and Xihui Liang*
Department of Mechanical Engineering, University of Manitoba, Winnipeg, Manitoba, Canada
* Xihui.Liang@umanitoba.ca

ABSTRACT

A practical product reliability assurance is an important task for all manufacturing lines. During recent decades, different approaches have been suggested for the reliability assessment of structures. However, most of them are not accurate enough, especially while dealing with stochastic and nonlinear deteriorating structures. Recently, the Probability Density Evolution Method (PDEM) have been introduced to overcome such shortcomings in the field of reliability calculations. But, it has been proven in previous researches that the PDEM needs some improvements in order to become applicable for practical product reliability assurance purposes. Accordingly, this paper offers a novel approach for product lifecycle reliability estimation using a limited number of evaluation tests. On this basis, PDEM is utilized together with an extreme learning machine (ELM) network in order to provide a time-varying reliability estimation of different structural systems under stochastic manufacturing, loading and environmental condition. After introducing a proper procedure for the proposed reliability prediction scheme, a set of hundred similar composite beams were utilized for experimental verification and validation tests. The results reveal that the proposed method significantly improves the performance of the PDEM method. The outcome of this research paves the way of manufacturing reliable products while minimizing quality and product assurance efforts

KEYWORDS: Structural Reliability, Probability Density Evolution, Extreme Learning Machine

1 INTRODUCTION

Strengthening the methods of evaluation of product reliability after manufacturing is gradually gaining manufacturers' attention. It is practically impossible to avoid variability in the production process. In this context, different reliability evaluation methods have been proposed such as environmental stress screening (ESS) or Accelerated Stress Screening (ASS). However, these methods, are requiring accurate system models or some expensive experiments (e.g. destructive tests) or they are needing all manufactured items to be tested. Accordingly, an effective reliability estimation tool based on smart learning algorithms can pave the way to improve product reliability at a minimum cost. In this regard, here the utility of an extreme learning machine (ELM) approach together with Probability Density Evolution Method (PDEM) is proposed.

The applicability of the PDEM for reliability assessment has been developed to a proper level of maturity during the last decade and it has been also experimentally evaluated (S. Saraygord Afshari and Pourtakdoust 2018). A shortcoming of this method for product reliability prediction and further reliability assurance is its relatively less accuracy while taking limited samples for reliability prediction. Accordingly, here we are introducing an ELM approach for enhancing the PDEM accuracy

In order to effectively utilize the PDEM for reliability assurance of products, there are two main deficiencies that should be resolved: firstly, the product degradation vs time and secondly the process of sampling in order to tune the PDEM equation for more accurate results. For solving such challenges, Artificial Neural Networks (ANNs) based modelling approaches have been applied in many references for improving predictive models. In this sense, an extreme learning machine (ELM) as an upgraded ANN has received particular attention in recent years (Xu et al. 2018). ELM can be effectively utilized for feature learning and clustering which helps us to improve the PDEM using an optimal set of experiments during the manufacturing phase. Hence, a hybrid simultaneous utilization of the PDEM and ELM will result in a fast and course reliability assurance procedure, which is a significant benefit in modern manufacturing schemes.

2 PROBABILITY DENSITY EVOLUTION METHOD

PDEM provides a valid time-varying joint probability density function (PDF) of the uncertain parameters of a physical system. The formulation of the PDEM is as follows (Li and Chen 2004).

2.1 PDEM Formulation

a general multi-degree-of freedom dynamical system can be stated as follows:

$$\mathbf{M(\Theta)\ddot{X} + C(\Theta)\dot{x} + f(\Theta, X) = F(\Theta}, t) \tag{1}$$

where $\mathbf{\Theta}$ is the representative vector of all random parameters, i.e. the physical and environmental uncertain parameters. **F** is the nonlinear restoring force, and $\ddot{X}, \dot{X}$ & X are the acceleration, velocity and displacement vectors of order N respectively.

It should be noted that as the PDEM is usually going to be utilized with a sensor data and the sensor data may be a result of several dynamic response measurements, we can define a Z(t) vector of the monitoring variable, which is evidently can be represented as follows:

$$\mathbf{Z}(t) = \Psi[\mathbf{X}(t), \dot{\mathbf{X}}(t)] \tag{2}$$

Here, Ψ is an operator which converts the state vectors into the measurement quantity, e.g. the structural strain at a critical point. It is clear that the response, Z(t) is a function of $\mathbf{\Theta}$. Accordingly, we can write Z(t) and its derivative as follows:

$$\mathbf{Z}(t) = \mathbf{H}_{\mathbf{Z}}(\Theta, t), \dot{\mathbf{Z}} = \mathbf{h}_{\mathbf{Z}}(\Theta, t) \tag{3}$$

Having these definitions for the measurement, its derivative, and the uncertainty vector, using some mathematical derivations together with the principal of probability preservation (Li and Chen 2004), a partial differential equation can be resulted for calculating an evolutionary joint probability density of Z and $\mathbf{\Theta}$, i.e. $p_{\mathbf{Z}\Theta}(\mathbf{Z}, \Theta, t)$. This partial differential equation is called the PDEM equation and it can be written as follows (Li and Chen 2004):

$$\frac{\partial p_{\mathbf{Z}\Theta}(z, \theta, t)}{\partial z} + \dot{Z}(\theta, t) \frac{\partial p_{\mathbf{Z}\Theta}(z, \theta, t)}{\partial t} = 0 \tag{4}$$

I.Cs: $\partial p_{\mathbf{Z}\Theta}(z, \theta, t)\,|_{t=t_0} = \delta(\mathbf{z} - \mathbf{z}_0) p_{\Theta}(\theta)$

Here, z_0, denotes the initial vector which is a deterministic value. At this point, two different approaches can be taken in order to find the joint pdf using Eq.5. First, one can perform an experiment to grab a time-response data of the uncertain system and the solve the PDEM equation using the $\dot{\mathbf{Z}}$ which is directly measured from sensor data, we call this approach the online PDEM. The second approach is to find $\dot{\mathbf{Z}}$ by solving the system equation, i.e. Eq.2, we call this approach the offline PDEM. In both offline and online approaches, after solving the PDEM equation (Eq.5), the PDF of Z(t) can be concluded by a simple integration over the domain of variation of θ:

$$p_Z(\mathbf{Z},t) = \int_{\Omega\mathbf{\Theta}} p_{\mathbf{Z\Theta}}(\mathbf{Z},\boldsymbol{\theta},t)\mathbf{d\theta} \tag{5}$$

2.2 PDEM-Based Reliability Calculation

Thus far, the calculation of an evolutionary estimation of the measurement (physical properties) is described and $p_Z(\mathbf{Z})$ is calculated vs time. Now, having a certain definition of the safe domain of operation, one can perform another integration over the safe domain to find the time-varying reliability of the system:

$$R(t) = p(\mathbf{Z})|_{\Omega s} = \int_{\Omega s} p_{\mathbf{Z}}(\mathbf{Z},t)\mathbf{dZ} \tag{6}$$

2.3 Numerical Algorithm for Structural Reliability Analysis

The steps for the PDEM-based reliability assessment is displayed in Figure 1 and detailed as follows:

1. Discretize the stochastic parameters vector.
2. For each discretized element of the parameter vector, identify the physical parameters.
3. calculate the velocity by solving Eq. (1) through any deterministic numerical/analytical method;
4. Substitute the calculated velocity into Eq. (4) and solve the PDE in order to find the evolutionary joint probability density.
5. Repeat the previous steps to find the joint PDF at all discretized points.
6. Use the results of step 5 to achieve the evolutionary PDF of **Z** through the discretized form of Eq. (5), Using the result of step 6, Eq. (6) integrate the PDF over the safe domain of operation in order to find the time-varying reliability.

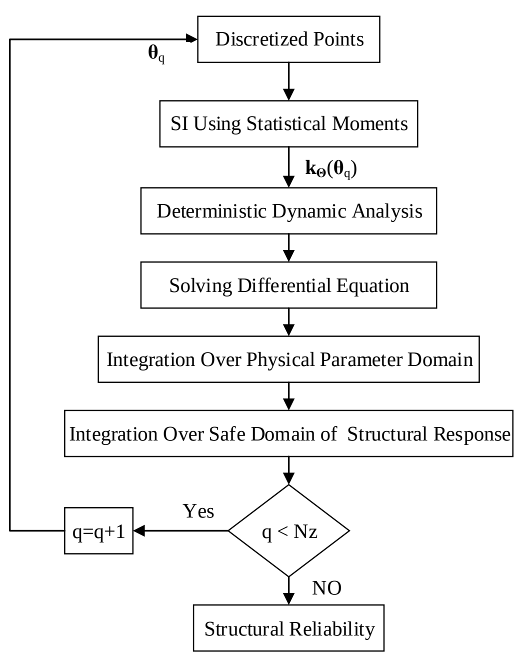

Figure 1. The numerical procedure of the PDEM-based Reliability Assessment (S. Saraygord Afshari and Pourtakdoust 2018)

Now we can distinguish three different approaches for reliability assessment: 1-online method 2-offline method 3-experimental method. For the sake of more clarity, consider a set of 100 samples in a manufacturing line. For achieving experimental time-varying reliability, it is required to perform 100 tests for the whole time interval of interest. For online time-varying reliability, only 1 sample has to be chosen and prepared for the test, again for the whole duration of the expected performance. As it is evident, it is definitely hard, time-consuming and laborious to perform experimental reliability tests, and this is the main constraint which makes the product reliability assurance impossible in some cases. In this context, the use of offline/online PDEM method is a great step forward for a fast and course reliability assurance process. However, it has been studied in the literature that the PDEM-based reliability assessment method is not accurate in some aspects, especially for the offline case. Here, an ELM is utilized to correct the trend of PDEM-based reliability curves so as to make it as close as possible to the experimental curve.

3 EXTREME LEARNING MACHINE

As demonstrated in Fig 2, only one hidden layer is considered in our ELM network. In this network, the weights between the green and yellow are generated as random variables. Assuming N dataset for the network, one can write the error cost function of ELM as follows:

$$E = \left\| H_0 \beta - Y_0 \right\| \tag{8}$$

the H_0 in Eq.8 is the outputs of the yellow neurons, β is the weight between yellow and blue neurons and Y_0 is the desired trained output. The main Phases of the ELM can be summarized as follows:

1) Randomly generation and allocation of the first set of weights a_i and bias b_i .
2) Hidden layer output calculation.

$$H_0 = \begin{bmatrix} g(a_1, b_1, x_1) & \cdots & g(a_{\tilde{N}}, b_{\tilde{N}}, x_1) \\ \cdots & \ddots & \cdots \\ g(a_1, b_1, x_{N_0}) & \cdots & g(a_{\tilde{N}}, b_{\tilde{N}}, x_{N_0}) \end{bmatrix}_{N_0 \times \tilde{N}} \tag{9}$$

3) Calculation of the ELM output (T_0).

$$T_0 = \begin{bmatrix} t_1^T \\ \vdots \\ t_{N_0}^T \end{bmatrix}_{N_0 \times M} \tag{10}$$

4) Output weights calculation

$$\beta^0 = P_0 H_0^T T_0 \; ; P_0 = (H_0^T H_0)^{-1} \tag{10}$$

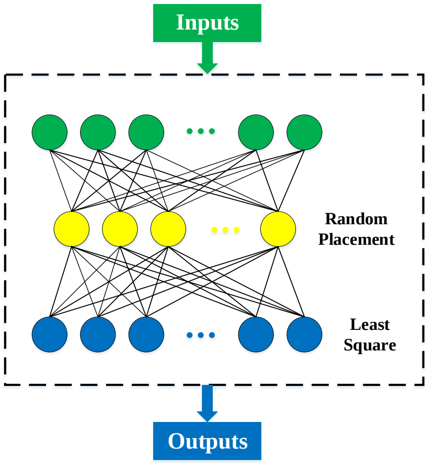

Figure 2. General scheme of an ELM assembly

The ELM output here in our PDEM-based reliability assessment, are the updated physical characteristics of the structure and the inputs are the primary values of the physical parameters. In this paper, the network is solved by a model-in-the-loop technique using TCACS optimization method which is the combination of Continuous Ant Colony System (CACS) and Tabu Search (TS) algorithms. Further details about the TCACS algorithm is briefly described in (Nobahari, Hosseini Kordkheili, and Afshari 2014).

4 HYBRID ELM-PDEM FOR RELIABILITY PREDICTION

As it is stated before, the PDEM is a strong tool for updating the probability function of uncertain parameters of different stochastic dynamical systems. However, it needs to be tuned in order to be utilized in an offline/online manner. The online (SHM-based) reliability assessment methods demonstrated an acceptable matching with the experimentally calculated reliability curve. Nonetheless, the offline approach was not performed well enough to be utilized as a reliability assurance reference (S. Saraygord Afshari and Pourtakdoust 2018).

Hence, in this research, the online curve of the PDEM-based reliability is taken as the reference output for training the ELM network. In other words, offline reliabilities of a set of samples are calculated and utilized as the input of the ELM. The ELM algorithm starts to train in a way to generate the online PDEM-based reliability curve using the multiple offline PDEM-based reliability curves. In order to optimize the training accuracy, a cost function using the Integral of Absolute Error (IAE) is defined to minimize the deviation of the ELM output, $R_{i_{of}}(t)$ from the desired online PDEM curve, $R_{i_{on}}(t)$. The definition of this IAE optimization cost is as follows.

$$IAE = \int_0^{t_f} \left| R_{i_{on}}(t) - R_{i_{of}}(t) \right| dt \tag{9}$$

5 CASE STUDY: VIBRATING COMPOSITE BEAM

In order to validate and verify the efficiency of the presented method, a simple cantilevered composite beam is chosen as a stochastic structure under stochastic loading. The online, offline and experimental PDEM-based reliability curves will be calculated for the presented beam structure and the hybrid ELM-PDEM reliability curve will be also plotted to make basic comparisons.

5.1 Experimental Setup

A hundred samples of a composite beam which with 65% fiber content in volume, is prepared for the experimental reliability test setup. As demonstrated in Figure 4, 2 piezoceramic patches are bonded on the beam, one for sensing the other one for applying the disturbance excitation force to the system. The piezoceramic patches and the beam thicknesses are 0.6 mm and 2 mm, respectively. Other dimensions are provided in figure 4, and a general view of the experiment is illustrated in Figure 3. In this experiment, both piezoelectric patches are attached to the beam root, since it is the best option for both the sensor and the actuator to observe and excite the preferred modes. Here, the piezoceramic actuator is vibrating using a PIEZO SYSTEM INC. 20X amplifier, where a PicoScope® 5000 Series data logger is used as an analog/digital converter.

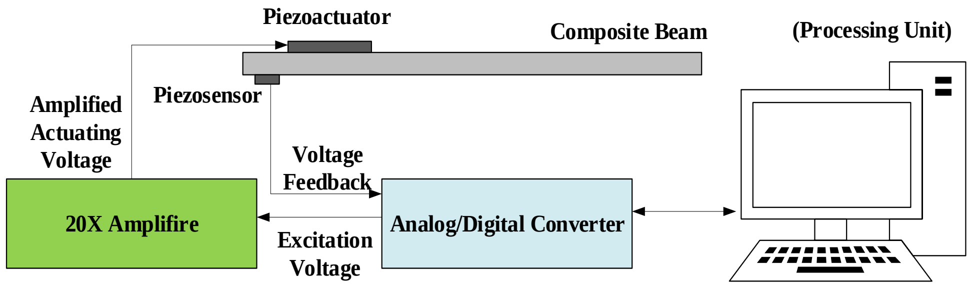

Figure 3. Schematic of the evaluation tests setup

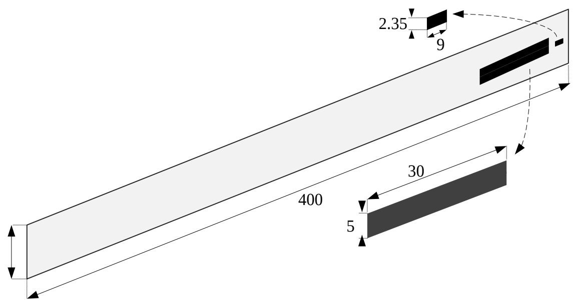

Figure 4. Elements of the experimental test setup (S. Saraygord Afshari and Pourtakdoust 2018)

5.2 Experimental Evaluation of the Hybrid ELM-PDEM reliability assessment

The current research purposes a manufacturing quality assurance based on the product reliability assurance. Consequently, it is essential to evaluate the precision of the estimated reliability using ELM-PDEM. Therefore, an accelerated test is applied to experimentally calculate the structural reliability of the cantilevered sample beam. In this regard, loading, failure criteria, and all test specifications were intended in a way to accelerate failure modes of interest while repeating the desired performance.

For the experimental reliability, each sample is exposed to similar loading in the same environment and the time responses of samples are gathered for 120 seconds. The time responses of all the trials are shown in Figure 5. The red parts in the figure denote the samples that have surpassed the failure criteria (piezo-sensor feedback out of [-3.2, 3.2] Volts). The experimental reliability curve using 100 repetitive tests is demonstrated in Fig. 6 as a green solid curve.

It should be noted that as much as the number of tests and samples increases, a smoother curve can be concluded for the experimental reliability.

The online and offline PDEM-based reliability curves are also calculated and presented in Fig. 6. As it is evident, both curves are showing a similar trend, however, the offline reliability curve faces a noticeable fault especially in the later time. This difference between the online and offline PDEM approaches emanates from different causes e.g. material degradation and fatigue, uncertainties in the data monitoring system, environmental changes, etc. These changes will reflect on the online data and so the online PDEM-based reliability curve becomes considerably accurate as compared to the offline curve.

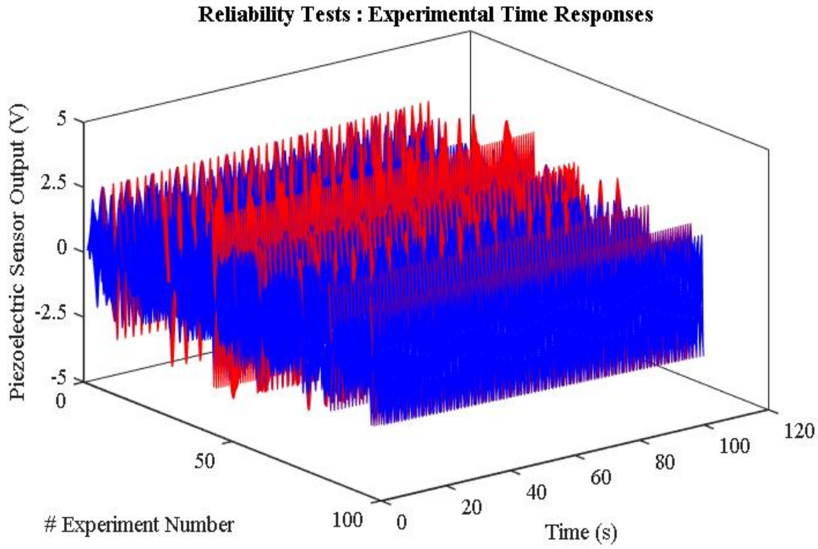

Figure 5. Time responses of 100 samples under excitation

For a manufacturing quality assurance based on product reliability, it is vital to find the most effective way to calculate the reliability with a lesser number of tests, but in an accurate way. Therefore, the experimental approach is not an amenable method. The online method is perfect for this objective. However, it is not always possible to perform a full lifecycle test in order to conclude an online curve. Hence the concept of hybrid ELM-PDEM, can make a powerful tool for a reliability assurance goal if it generates results similar to the online PDEM. The blue curve in Fig. 6, is the hybrid PDEM-ELM reliability curve which can be concluded using the identification of only one sample!

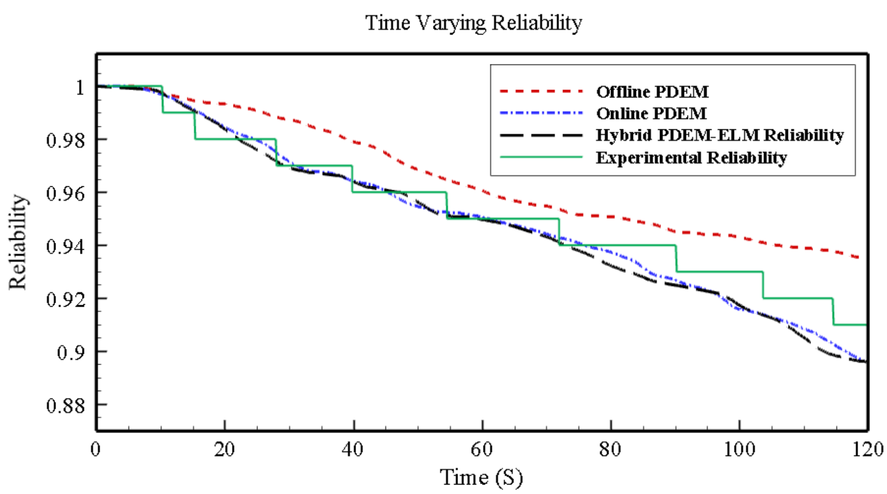

Figure 6. Time-varying reliability of the cantilevered beam using different experimental/numerical approaches

For the hybrid PDEM-ELM reliability curve in Fig. 6, ninety time responses out of the hundred experiments were used to train the ELM network and. The other ten time-responses were used for verification and plotting the curve in this figure.

6 CONCLUSION

An extreme learning machine approach for enhancement of probability density evolution method is presented in this paper. This approach is proposed in order to make it possible to find a proper estimation of the product reliability during its lifecycle using a limited number of experiments. The method and network training technique are presented as a hybrid approach for finding the most accurate reliability values vs time. A hundred cantilevered composite sample beams are also considered for performing reliability tests and further evaluations. Making comparisons between experimental reliability approach, offline, online and the newly proposed ELM-PDEM approach reveals that the new algorithm can significantly upgrade the results of the offline PDEM and hence it can be used as an effective means for product reliability assurance. Development of this method and applying it to more complex systems may meaningfully make a step forward in manufacturing quality assurance.

7 ACKNOWLEDGEMENTS

Research supported by the Natural Science and Engineering Research Council of Canada (RGPIN-2019-05361)

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