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EXPERIMENTAL, ANALYTICAL AND NUMERICAL STUDY OF BUCKLING OF CARDBOARD ANGLE

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ABSTRACT

This study aims to investigate the buckling behavior of cardboard angles. As a first step, the buckling is studied using experimental, analytical, and numerical approaches. The effects of cardboard angle geometry and boundary conditions are investigated. A custom set-up was designed to test angles under axial compression. Classical laminate theory based on Kirchhoff's hypothesis is used for the analytical study. The finite element method is used for the numerical approach. In contrast to the analytical approach, the numerical study accounted for out-of-plane shear strain. Results show that analytical results overestimate the buckling load compared to the numerical results. Also, the experimental results align more closely with the numerical model. This suggests that the effect of out-of-plane shear strain is important in cardboard angle under compression loading.

1 INTRODUCTION

Cardboard angles are packaging products used to protect edges during handling and storage. Most angles are laminated composite made from plies of paper, typically liners, that are glued together with a natural or synthetic adhesive, mostly dextrin or polyvinyl acetate, then folded into an L shape.
Paper is mainly composed of celluloses bonded together by hydrogen bonds. In-plane properties of paper are described following the machine direction and the cross direction, which refer to the direction of the flow of pulp in a Fourdrinier machine during the manufacturing process. In a laminated cardboard angle, all paper plies are in the machine direction due to the manufacturing process.
At a macroscale level, papers can be considered as orthotropic [1]. Due to the manufacturing process, out-of-plane properties, such as shear moduli and Poisson's ratios, are generally low compared to in-plane properties. Out-of-plane properties of laminated paper are known to be difficult to investigate. Some authors have tried to adapt test set-up developed for fiber-reinforced composites [2].
One of the primary loadings on angles is axial compression. There is currently no industry-accepted standard procedure for mechanical testing and characterization of cardboard angles. The lack of a standard procedure poses difficulties for manufacturers when benchmarking their products. To the best of our knowledge, the European standard EN 13393 [3] is the only standard to evaluate cardboard angle in compression. However, this standard doesn't take into account the various geometries of angles available on the market. Thus, concerns can be raised regarding the effect of buckling on the results, which can lead to difficulty in evaluating compression strength.
Buckling of columns and plates is an instability phenomenon that is greatly affected by boundary conditions and geometry. It can be observed by a transverse deflection and a reduction of the apparent stiffness under compression loading. For plates, buckling is mostly studied following three principal types of boundary conditions: simply

supported (S), clamped (C), and free (F) [4]. For analytical approach, the geometry of an angle can be simplified as a single plate where one lateral edge is simply supported, and the other is free [5].
The analytical buckling models based on classical lamination theory (CLT) assume Kirchhoff's hypothesis: the section of the plate remains straight and normal to the middle plane and the thickness is constant. This means that no shear or normal strain is taken into account through the thickness. This theory is suitable for thin plates when out-of-plane shear moduli are comparable to the in-plane shear modulus [6].
The first-order shear deformation theory (FSDT) was designed to consider the out-of-plane shear moduli. As for the CLT, the FSDT assumes that the section remains straight, but not necessarily normal to the middle plane [7]. This theory considered that out-of-plane shear strain is linear. It is suitable for thin to moderately thick plate. Finite element analysis using thick plate elements in Abaqus Software are in agreement with FSDT [8].
By having a good understanding of the conditions that initiate buckling of cardboard angles, it is assumed that it will be possible to design an acceptable standardized procedure to evaluate this type of product in axial compression.
The objective of this project is to study the buckling behavior of laminated cardboard angles under axial compression. To do so, analytical and numerical models were developed to study buckling following two sets of boundary conditions along the four edges: SSSF and CSCF. The first and third letters correspond to the boundary conditions at the extremities where the load is applied. These boundary conditions are considered to reflect usage conditions when the cardboard angles are used to support stacking merchandise. The analytical model was based on CLT, and the finite element method was used for the numerical model. Experimental testing was finally performed to validate both models. This project is done in collaboration with Abzac Canada Inc., a manufacturer of packaging products.

2 MATERIALS AND METHODOLOGY

2.1 Materials and manufacturing of specimens

A commercial recycled liner and polyvinyl acetate adhesive were used to manufacture all specimens. At a moisture content of 7 %, the liner has a thickness of 265,7 µm and a weight of 169,5 g/m2. The adhesive's solid content is 19,4 %. These materials are the most commonly used by Abzac Canada Inc. to manufacture angles.
Rectangular laminated plates were manufactured to determine the elastic properties. They were composed of 15 liners, each one oriented in the same direction and separated by a thin layer of adhesive. The adhesive was applied using a cylinder coater, and excess of adhesive was removed with a manual press roller. Perforated wood plates of 4,76 mm thickness were placed against the laminated plates to ensure straightness during drying for a minimum of 48 hours. A CNC machine Shapeoko Pro from Carbide 3D was then used to cut the rectangular plates into specimens with the desired dimensions. The table is equipped with a router bit Freud 76-102. The rotation and displacement speed of the bit were respectively set to 18 000 RPM and 12,7 mm/s.
Cardboard angles with a fixed width of 50 mm per spar were manufactured to study buckling experimentally. They were manufactured in a similar manner as rectangular plates but using a steel L shaped jig during drying to obtain the desired 90° angle. The angles were cut at the required length with a 180 teeth 10 inches electric miter saw.
All specimens were then conditioned at 50 % relative humidity and 23°C for a minimum of 48 hours before testing. After conditioning, the average thickness of a ply is 0,24 mm. It was calculated by dividing the average whole thickness of the laminates by the number of plies.

2.2 Determination of elastic mechanical properties of a ply

In-plane and out-of-plane elastic properties were obtained from the rectangular laminated plates specimens. The in-plane properties were required to study buckling with the analytical model. For the numerical model, the in-plane properties were required as well as the out-of-plane shear moduli. All the tests were performed using a universal testing device with a 50 kN load cell at a testing speed of 1 mm/s.

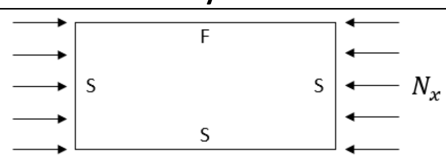
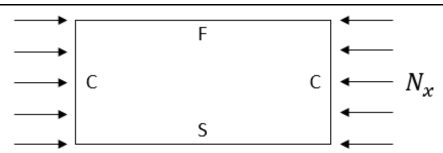
Tensile testing based on ASTM D3039 [9] was performed on specimens with dimensions of 25 x 250 mm. In-plane moduli (E_1 and E_2) and Poisson's ratio v_{12} were obtained using a mechanical extensometer and digital image correlation (DIC). For the in-plane shear modulus G_{12}, tensile tests were performed following a similar methodology on 45° specimens. The equivalent modulus $\bar{E}_x$ was measured and G_{12} was evaluated using CLT.

Out-of-plane shear modulus G_{13} and G_{23} were obtained through reverse engineering. Inspired by ISO 14130 [10], three-point bending tests were performed on short specimens with dimensions of 18 x 80 mm. The radius of the supports and the loading nose was 5 mm, and the span length was fixed at 30 mm. The short span length ensures that out-of-plane shear stress is important. The displacement of the loading nose was obtained using DIC. In Abaqus, a plate with 18 x 80 mm dimensions was modeled as a laminated orthotropic unidirectional plate composed of 15 plies. Shell elements S4R with 1 x 1 mm dimensions were used to mesh the laminate. These dimensions were chosen based on a convergence analysis which consisted of reducing the dimensions until the difference in the force reaction of the loading nose was less than 1 % of the previous iteration. The supports and loading nose were modeled as rigid elements. The experimental linear part of the force-displacement curve was then replicated in Abaqus software by varying the out-of-plane shear moduli until a good agreement was found.

2.3 Analytical study of buckling

The analytical study relies on approximate solutions based on CLT. Kollár and Springer [11] used the Ritz method to solve the equation for the deflection of plate as given by Whitney [12]. Approximated solutions for the buckling load $N_{x,cr}$ are presented in Table 1 where L_x and L_y are respectively the dimensions parallel and perpendicular to the loading N_x. The terms D_{ij} are the stiffness coefficients for an orthotropic and symmetric laminated plate. The sets of boundary conditions SSSF and CSCF were considered for a 15 plies laminate with various values of L_x and L_y.

Table 1 Boundary conditions and buckling load of unidirectionally loaded orthotropic and symmetric laminated plate [11].

	Boundary conditions	**Buckling load $N_{x,cr}$**
SSSF		$\frac{\pi^2 D_{11}}{L_x^2} + \frac{12 D_{66}}{L_y^2}$
CSCF		$\frac{\pi^2 D_{11}}{(0{,}5L_x)^2} + \frac{12 D_{66}}{L_y^2}$

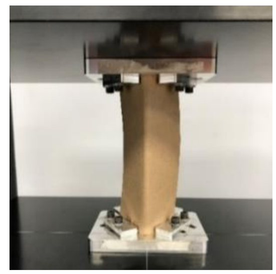
2.4 Numerical study of buckling

The numerical study was performed using Abaqus/Standard software. Angles were modeled as two laminated plates joined by the middle plane at a 90° angle. Both plates were modeled and meshed in the same manner as described in section 2.2. Translation and rotation at extremities were imposed according to the boundary conditions SSSF and CSCF. A material coordinate system was applied to both plates in order to assign material orientation. A uniformly distributed load was applied in the longitudinal direction on the upper edge of the angle. It is assumed that delamination doesn't occur at the initiation of the buckled state. Thus, cohesive elements for the adhesive were not considered. Finally, an eigenvalue buckling analysis was performed to evaluate the buckling load $N_{x,cr}$ for a 15 plies laminate with various values of L_x and L_y.

2.5 Experimental study of buckling

Experimental study of buckling was performed using a universal testing device with a load cell of 10 kN at a testing speed of 1 mm/min. Cardboard angles specimens were mounted at both extremities in a custom adjustable device to prevent sleeping and end crushing as illustrated in Figure 1. This device was designed to replicate clamped boundary conditions. The assembly was inserted between the two compressive plates of the testing device. The crosshead displacement and the load were recorded at a rate of 10 Hz during testing.

a)



b)

Figure 1 Experimental set-up: a) custom adjustable device and b) angle mounted assembly.

The experimental buckling load was evaluated using an offset method with a 0,05 mm offset of the linear part of the force-displacement curves. The buckling load was identified at the intersection of the offset curve and the actual curve. Angle length of 100, 150, 200, 250 and 300 mm were evaluated with three repetitions per length.

3 RESULT AND DISCUSSION

The elastic properties of a ply obtained with the tensile and three-point bending tests are presented in Table 2. The equivalent modulus $\bar{E}_x$, used to calculate the in-plane shear modulus G_{12}, was found to be 4010 MPa.

Table 2 Elastics properties of a ply.

E_1 (MPa)	E_2 (MPa)	G_{12} (MPa)	G_{13} (MPa)	G_{23} (MPa)	v
5360	3667	1500	41	37	0,31

Buckling load per unit length $N_{x,cr}$ calculated by the analytical and numerical model for various dimensions are presented in Figure 2 for SSSF and CSCF boundary conditions. Results are shown for a laminate consisting of 15 plies, with a total thickness of 3,6 mm. This thickness is common for cardboard angles.

Figure 3 shows the buckling load $N_{x,cr}$ obtained by experimental, analytical, and numerical approach for angles with a fixed width of L_y = 50 mm. This dimension corresponds to one of the most common widths for cardboard angles. Comparing both models under the given boundary conditions reveals that the analytical model significantly overestimates the buckling load for any dimensions. This suggests that out-of-plane shear strain has a great impact on the buckling behavior of cardboard angles. Moreover, both analytical and numerical CSCF models exhibit a higher buckling load compared to the SSSF boundary conditions in their respective model. This is due to the fact that degrees of freedom for CSCF boundary condition are reduced, thus needing a higher load to induce an out-of-plane deflection. Also, for both models, is it observed that the buckling load under CSCF boundary conditions is more sensitive to the variation of length L_x than under SSSF boundary conditions.

Although the experimental set-up was originally designed to replicate clamped boundary condition, the experimental results fall between CSCF and SSSF boundary conditions of the numerical models and are closer to the

SSSF case. Both numerical and analytical models don't account for geometrical imperfection and assumed a uniformly distributed load. Despite efforts to ensure straightness of specimens using a jig, visual inspection reveals that they were not perfectly straight. Also, it was difficult to cut angles in a way that ensures parallelism of both extremities. Thus, a uniformly distributed load was not guaranteed.

Overall, the results are satisfactory in an industrial context, but some improvements can be brought to the experimental approach. One modification could involve a better way to cut extremities of the angles to ensure parallelism of both edges, removing the worry about the distribution of the load. Also, the use of tabs could reduce the stress concentrations. Finally, DIC could be used to evaluate more accurately the initiation of the buckled state.

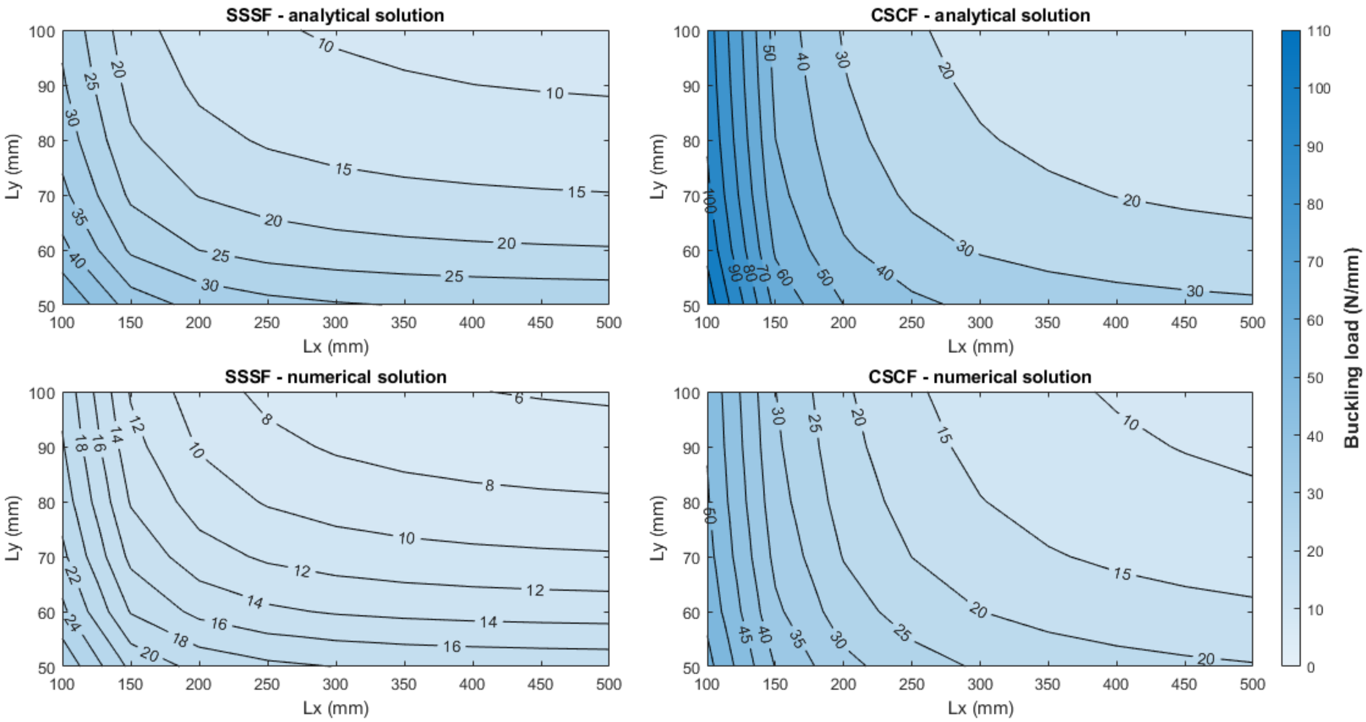


Figure 2 Buckling load for a 15 plies laminate obtained by analytical and numerical study for SSSF and CSCF boundary conditions.

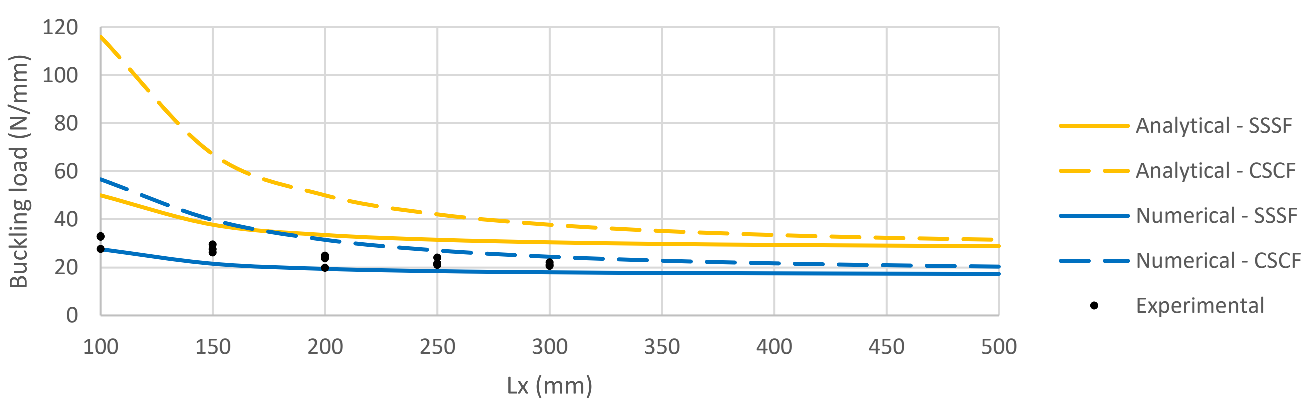


Figure 3 Experimental, analytical, and numerical buckling load for a 15 plies cardboard angle with L_y = 50 mm.

4 CONCLUSION

The buckling behavior of laminated cardboard angles was studied by an analytical, numerical and experimental approach. In order to do so, elastic properties of laminated papers were first obtained by experimental testing combined with finite element simulations. The properties were used in the analytical and numerical model. The analytical buckling model is based on the classical laminate theory and only consider in-plane properties. The numerical buckling model used finite element method. This model considers in-plane properties as well as out-of-plane shear moduli. Clamped and simply supported boundary conditions at the extremities of angles were studied with these models. Experimental study of buckling was carried on using a custom set-up that replicates clamped boundary conditions on a cardboard angle for which each spar is akin to a plate with one long edge being free and the other simply supported (along the spine). An axial compression load was applied to specimens until buckling occurred. The results show that the analytical model greatly overestimated buckling load compared to the numerical model for both boundary conditions. This means that out-of-plane shear strain can't be neglected for laminated papers. Experimental results are closer to the numerical model for a simply supported boundary condition. The experimental results could be improved by modifying the experimental set-up to allow for a better uniformly distributed load and by evaluating the buckling load with the help of digital image correlation method.

5 ACKNOWLEDGEMENT

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