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OPTIMIZATION OF FIBRE ORIENTATION WITH A CONTOUR-BASED FIBRE MAPPING METHOD

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ABSTRACT

Curved fibres in fibre reinforced composites are being used more frequently due to the development of advanced manufacturing and analysis techniques. Using curved fibres improves the elastic properties of composites, particularly their stiffness distribution, compared with straight fibres. To exploit the potential of curved fibres, many fibre orientation optimization methods have been proposed. In this paper, a contour-based fibre mapping (CBFM) method is explored to represent fibre paths by the two-dimensional contours of a three-dimensional generating function. The generating function consists of the sum of radial basis functions (RBFs) that produce curved fibres and a linear function, which is an oblique plane, that produces straight fibres. The contours, and hence the fibres, are smooth, continuous and require minimal post-processing. The fibre orientation is updated by following the gradients of the objective, stiffness, with respect to the parameters of the RBFs, which are the design variables. One example, a cantilever subject to bending, is discussed to demonstrate the capability of the CBFM method to optimize fibre patterns efficiently.

1 INTRODUCTION

Due to the superior performance of carbon fibres as light-weight structures compared with conventional metallic alloys, the demand for composite materials in many fields is increasing. Carbon fibre reinforced polymers (CFRPs) have been widely used in the aerospace, automotive and marine industries [1]. With the development of advanced manufacturing technologies, tailoring of fibre directions is more accessible. Automatic fibre placement (AFP) allows fibres to be positioned at nearly arbitrary angles [2]. Variable stiffness design has been conducted on different composite structures for multiple optimization purposes, such as improving stiffness and maximizing buckling load. Studies indicate that a well-designed fibre configuration can significantly enhance the mechanical performance of fibre reinforced composites [3–5].

The first step in fibre orientation design is to find an appropriate method to parameterize fibre paths. A commonly adopted method is to define fibre orientations at discrete points [6] using the finite element method (FEM). In each element, the fibres are considered locally straight. This discrete method is mathematically convenient but computationally expensive if the number of elements is large, because the fibre angle in each element is a design variable. Moreover, the resulting fibre paths are not continuous, which means further post-processing is needed. Alternatively, continuous fibre parameterization approaches guarantee fibre continuity and improve computational efficiency. Continuous fibres can be modelled using analytical functions [7], such as spline curves, trigonometric function, cubic NURBS curves [8] and other functions. Other implicit functions using lamination parameters [9] and streamlines [10] also generate continuous fibres in the optimization. Continuous methods typically require fewer design variables compared to discrete methods, resulting in lower computational costs. However, the selection of a reference path severely constrains the possible fibre configurations that can be represented.

Based on studies of discrete and continuous fibres, it is appealing to incorporate the merits of both methods. Arbitrary fields of fibres, including straight and curved ones, should be generated by a compact method while guaranteeing fibre continuity. The method should have the capability to generate complex configurations with a simple model. One potential approach is to use the two-dimensional contours of three-dimensional functions to represent fibre paths. The contours generated are inherently continuous, which is ideal for representing curved fibres. The parameters of the higher dimensional functions control the configuration of fibres and are the design variables in optimization. The idea originates from the level set method [11], and is studied using different forms of generating functions. Honda and Narita [12] use a cubic polynomial function and its contours to represent curvilinear fibres, in order to maximize the fundamental frequency of a laminated composite plate. A set of equally spaced fibres can be generated by different constant level set values, which is proposed to optimize fibre orientations for minimizing compliance by Brampton et al. [13]. Tian et al. [14] develop a parametric divergence-free method to describe the fibre angle arrangement using over a thousand basis functions.

A contour-based fibre mapping (CBFM) method is proposed in this paper, which uses a three-dimensional function to represent fibres in a two-dimensional space through the summation of several compactly supported radial basis functions (CS-RBFs) and an oblique plane to generate both straight and curved fibres. The fibre configuration is optimized by updating shape parameters in the generating function using a gradient descent method. A common optimization case is then studied using this method.

2 Contour-based Fibre Mapping

2.1 Generating function

This CBFM method is based on the level-set method, which depends upon a generating function in three dimensions (*x*-*y*-*z*) and its projection on two dimensions. The contours are considered as fibre paths for their smooth and continuous characteristics. The generating function ψ is a linear summation of several basis functions ϕ with shape parameters A:

$$\psi(\boldsymbol{x}) = \sum_{i=1}^{N} A_i \phi_i(\boldsymbol{x}), \tag{1}$$

where ϕ_i is the i-th basis function of total number N, $\boldsymbol{x} = (x, y)$ are the coordinates of any points in the design domain, and A_i is the shape parameter that changes the amplitudes of basis functions. Many choices of basis functions are possible; here, radial basis functions are selected for their smoothness and facility in representing many other types of functions. The value of a radial basis function at $\boldsymbol{x}$ is

$$\phi_i(\boldsymbol{x}) = \phi_i(||\boldsymbol{x} - \boldsymbol{x_i}||), \tag{2}$$

which is determined by its distance to the centre of basis function $\boldsymbol{x_i} = (x_i, y_i)$. The distance in the $x - y$ plane is $R_i = \left||\boldsymbol{x} - \boldsymbol{x_i}|\right| = \sqrt{(x - x_i)^2 + (y - y_i)^2}$.

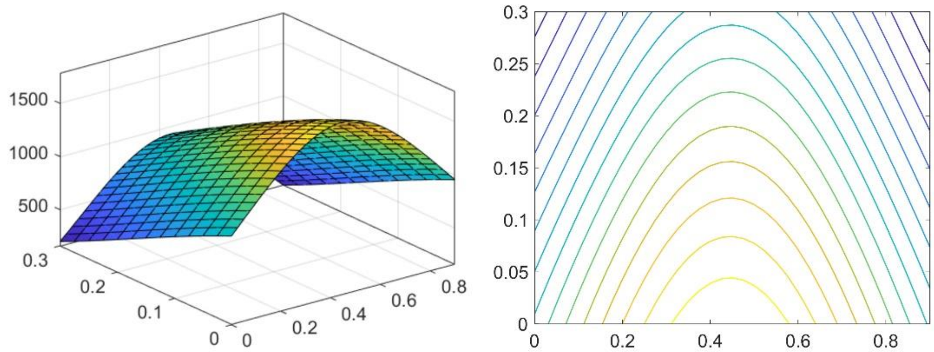


Figure 1. Example of a function in three dimensions and its contour in two dimensions.

A compactly supported RBF from Wendland [15] is chosen for the construction of such basis function. The change of one basis function will only influence a limited region, which tends to improve the convergence in optimization. Wendland's compactly supported RBF is

$$\phi_i(\boldsymbol{x}) = \begin{cases} \left(1-\frac{R_i}{W_i}\right)^4 \left(4\frac{R_i}{W_i}+1\right), & \text{if } \frac{R_i}{W_i} \leq 1 \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

where W_i is the width of radial basis function i. This formulation ensures that the change in one basis function has no influence outside its local region. Here the width W_i is fixed for each function and only the amplitudes A_i are design variables.

2.2 Fibre orientation

The configuration of fibre contours is determined by the design parameters in the generating function. To find the optimal fibre configuration, the relationship between the design parameters and contours is required. The design space is discretized into elements with FEM and the fibre angle at the centre of each element is the element angle, as shown in Figure 2. The fibre angle at an arbitrary point (x_c, y_c) in the domain of the j-th element is computed using the gradients of the generating function:

$$\theta_j = \arctan\left(-\frac{\partial\psi}{\partial x} / \frac{\partial\psi}{\partial y}\right). \tag{4}$$

The fibre angle is between $(-\frac{\pi}{2}, \frac{\pi}{2}]$. This angle is used in calculating the elastic properties of the composite and the sensitivity analysis during optimization.

3 Optimization

3.1 Objective

To improve the mechanical properties of fibre reinforced composites, the in-plane stiffness is maximized with fibre orientation optimization. In other words, the purpose of the optimization is to minimize the compliance of a composite laminate. For a structure in equilibrium, define the compliance as:

$$c = f^T d = d^T K d, \tag{5}$$

where f is the load vector, d is the nodal displacement vector and K is the global stiffness matrix.

3.2 Initialization

The first step in optimization is to select an initial guess for the design variables. It has been shown that a good starting point for gradient-based optimization improves the optimization efficiency [16]. It is known that the alignment of fibre directions and local principle stress directions reduces compliance [7]. The initialization of the fibre pattern is also the initialization of the design parameters, which are the location of RBF centres and the amplitudes of the RBFs. A simple method is to place RBFs in grids with uniform spacing and equal amplitudes. This method is straightforward and easy to implement, but the initial configuration is not ideal for many cases. An oblique plane ψ_0 is introduced to generate unidirectional fibres as initial patterns while setting all RBF amplitudes to zero. The generating function is

$$\psi = \psi_0 + \sum_{i=1}^{N} A_i \phi_i(\boldsymbol{x}) = x * \tan(\theta_0) - y + \sum_{i=1}^{N} A_i \phi_i(\boldsymbol{x}). \tag{6}$$

θ_0 is the angle of unidirectional fibres. The introduction of unidirectional fibres as an initial guess has many advantages. The amplitudes of the radial basis functions can initially be zeros, without extra computational cost. For many cases, a configuration with straight fibres is close to the optimal solution and will provide fast convergence. The oblique plane both scales the solution and provides uniqueness; without the oblique plane linearly scaling all the RBF amplitudes by the same value would generate the same contour pattern. In the following optimization, the enhanced generating function is applied.

3.3 Procedure

The optimization is:

1. Initialize the fibre configuration with unidirectional fibres of angle θ_0 and amplitudes $A_i = 0$.
2. Perform finite element analysis with an anisotropic composite model.
3. Compute the sensitivity of the compliance with respect to the design variables A_i.
4. Update A_i and the fibre angle θ_j in each element.
5. Check if the stopping criteria is satisfied. If not, return to step 2 and continue.
6. Obtain the optimized fibre configuration.

4 Numerical Results

A common structural optimization case that contains bending and shear is a cantilever beam with the left side fixed and a load applied on the top right, shown in Figure 3. The mechanical properties of the composite are $E_1 = 150$Gpa,

$E_2 = 20$Gpa, $G_{12} = 7$Gpa, $\nu_{12} = 0.3$, based on the properties of common carbon fibre composites. The initial fibre angle is $1°$ after searching for a range of angles from 0 to $180°$ that provides the lowest compliance.

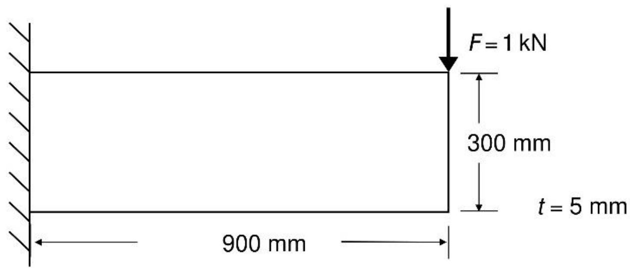
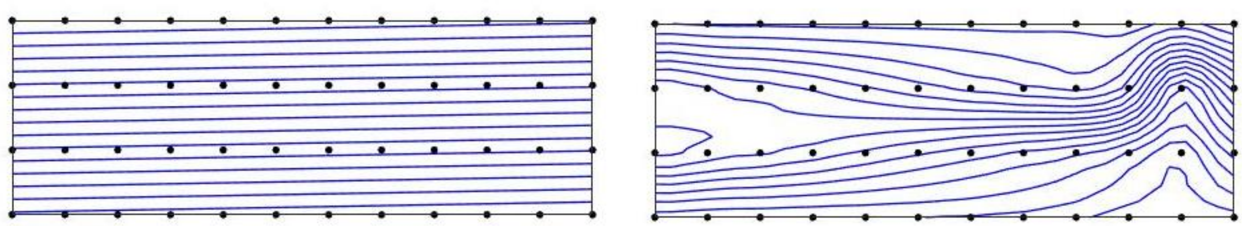


Figure 2. Load and boundary conditions of a cantilever beam.

Figure 3 shows the optimization result for the cantilever beam using 48 RBFs placed uniformly. The initial fibre pattern is represented in (a) by unidirectional fibres. After optimization, curved fibres appear in (b). The curvilinear fibres reduce the compliance 19% from 0.2914 (Nm) to 0.2359 (Nm). Compared to the same structure but with isotropic material, which has a compliance of 0.3888 (Nm), variable stiffness composite structures improve the mechanical properties.

The optimization result verifies the capability of the CBFM method to generate complex and physically relevant fibre configurations, including unidirectional fibres and curved fibres. With a small number of design variables, the compliance is minimized after 306 iterations. The continuous and smooth fibres are favorable for manufacturing with proper post-processing. Additional constraints to avoid gaps and overlaps will be discussed in future work.



(a) (b)

Figure 3. The optimization result of a cantilever beam after 306 iterations (50 seconds). (a) is the initial fibre pattern with $1°$ fibres, and (b) is the optimized contour with compliance reduced by 19%. The blue lines are fibre contours, and black dots are the locations of RBF centres.

5 Conclusion

A contour-based fibre mapping method is proposed to ensure fibre continuity and computation efficiency by using a generating function with shape parameters to adjust fibre configurations. The function consists of an oblique plane to generate straight fibres and radial basis functions to generate curved fibres locally. The contours of the higher dimensional function correspond to the fibre paths in a two-dimensional plane. A gradient descent algorithm is used to update the design variables, which are the amplitudes of the basis functions.

The fibre configuration is updated by adjusting the design parameters in the generating function, which reduces the amount of design variables compared to discrete methods, thus improving computational efficiency. The optimization result of a common cantilever beam shows that CBFM method is capable to generate both straight and curved fibres. Furthermore, this method enables the optimized fibre patterns to improve stiffness compared to the same structure with isotropic materials or straight fibres. The smoothness and continuity of contours are ideal for representing fibres without substantial post-processing, and manufacturing constraints will be added later to avoid defects, such as gaps and overlaps.

6 Reference

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