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APPLICATION OF FLEXURE TESTING FOR DETERMINATION OF TENSILE STRENGTH OF GFRP BARS

Lochan, Philip P.[1] **and** **Polak, Maria Anna**[1*]
[1] Civil and Environmental Engineering, University of Waterloo, Waterloo, Canada
* polak@uwaterloo.ca

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ABSTRACT

Tensile strength of GFRP bars is a primary property needed for the design of concrete structural members reinforced with these bars. However, tensile testing requires substantial time and laboratory resources. This paper aims to indicate the adequacy of a flexural test to assess the tensile strength from observing the rupture modulus, as it requires much less effort to conduct the test. Since calculations from existing ASTM standards cannot be accurately used due to the bi-moduli behaviour of GFRP, a different approach to relate the rupture modulus to the tensile strength is presented, which relies on concepts of beam mechanics, the stress-strain relationships, and the Weibull "weakest link" model. The experimental results for M8, M13, M16, M20 and M32 GFRP bars undergoing a three-point bending are compared to the selected direct tensile testing results. This study shows that the GFRP bars follow the "weakest link" model, where their strength is decreased as their size increases. Collectively, results from this study indicate flexural tests have great potential to be used as quality testing method for GFRP reinforcing bars.

1 INTRODUCTION

Glass Fibre Reinforced Polymer (GFRP) reinforcing bars is an alternative to the conventional steel rebar used in structural concrete elements. Their cost is becoming more comparable to steel rebar, while providing a higher tensile strength and corrosion resistance in the presence of moisture and chlorides in a concrete structure. However, because GFRP is a brittle material, quality control testing of GFRP bars is imperative prior to their use.

Tensile strength is a primary property required from reinforcing bars in structural concrete. Tensile testing for GFRP bars is typically completed using a uniaxial direct tension test, as defined in existing testing standards such as CSA S806-12 (Annex C) and ASTM D7205/D7205M-06. Using this type of test for GFRP bars is quite cumbersome to setup requiring a lot of specimen preparation prior to testing, namely casting the ends of GFRP bars into the appropriate grips/anchorages so that the specimen can be placed and pulled in grips of a testing machine. In addition, the specimens need to be long and are heavy following the specifications of testing standards (CSA S806-12; ASTM D7205). The larger the bar diameter, the longer the specimen; most laboratories do not have space and appropriate frames for such testing.

An alternative to the direct tensile test can be obtaining a tensile strength in GFRP bars using a flexural test. Since a beam-element is subjected to compressive and tensile stresses while bending, the behaviour near failure of the element can be observed and analyzed to help look at the tensile strength of a material. Setting up and conducting this test is quick and efficient since specimens do not require difficult preparation, and a low capacity machine (up to a load of no more than 25 kN) with the appropriate testing apparatus for 3-point-bending can be used to conduct the testing (Arczewska et al. 2019).

The goal of this paper is to outline the methodology of measuring the rupture modulus from flexural failure, and correlating it back to the actual tensile strength of a GFRP bar. This is achieved using equations and relationships from solid mechanics, utilizing the bi-modular behaviour of the GFRP material, and its failure distribution that follows Weibull's "Weakest Link" model. This work is being built upon the findings by Arczewska et al. (2019). Arczewska's scope of research only observed GFRP bars of 12mm (M12) and 16mm (M16) in diameter, whereas this paper presents results from GFRP bars of 8 mm (M8), 13mm (M13), 16mm (M16), 20mm (M20) and 32mm (M32).

2 BACKGROUND

2.1 Bi-Moduli Behaviour and Flexural Calculations

The proposed testing is based on ASTM D4476-14: Standard Test Method for Flexural Properties of Fiber Reinforced Pultruded Plastic Rods. It requires the specimen to be cut longitudinally, so that the specimen can fail in tension before its compressive stress capacity is reached. The procedure provides calculations and relationships for obtaining the stress, strain and corresponding modulus of rupture (tensile strength in flexure).

As a result of the bi-modular behaviour of the GFRP material (i.e. different behaviour in tension and compression), the relationships defined in ASTM D4476-14 cannot be directly utilized. (Arczewska et al. 2019). Since the specimen is subjected to tensile and compressive stresses when undergoing bending, equilibrium of forces and moments can be used along with stress-strain relationships represented as Equations [1] to [3] for calculations (Beer et al. 2012). Figure 1 provides a graphical representation for the derivation for these equations.

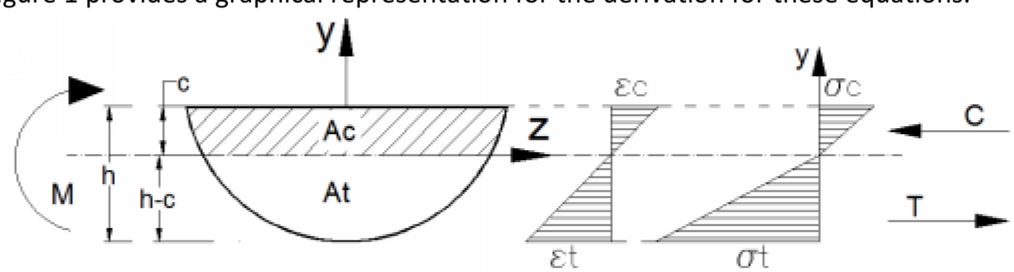

Figure 1: Distribution of bending stresses and strains of GFRP material along height of cross-section (Arczewska 2017).

$$\int_{A_t} \sigma_t \, dA + \int_{A_c} \sigma_c \, dA = 0 \quad [1]$$

$$\int_{A_t} \text{-}y\sigma_t \, dA + \int_{A_c} \text{-}y\sigma_c \, dA = M \quad [2]$$

$$\frac{\sigma_t}{E_t(h\text{-}c)} = \frac{\sigma_c}{E_c c} \quad [3]$$

where: σ_t and σ_c refers to tensile and compressive stress produced from bending, A_t and A_c refers to tensile and compressive areas from bending, E_t and E_c refer to the tensile and compressive elastic moduli, M represents the bending moment, h represents the height/depth of the specimen, and c represents the location of the neutral axis from the top surface of the specimen.

The three unknowns in the system of Equations [1] to [3] are the rupture tensile stress, compressive stress, and the location of the neutral axis of the specimen. The height, h, the radius r, and the length of the specimen are all measured prior to testing. The elastic moduli should be also measured or can be adopted from the GFRP manufacturer's data. The more commonly measured modulus is elastic modulus in tension E_t. The modulus in compression E_c can be measured (Arczewska et al. 2019), or a typical ratio (around 1.2-1.25) between tensile and compressive elastic moduli can be used (Jones 1977, 1978; Savchenko 2005). In this work, the results for compressive and tensile moduli measured by Arczewska et al. (2019) for similar bars are used. It should be noted that the final values for rupture modulus depend on the ratio of E_t/E_c and not on actual value of the moduli. After completing the integration due to the non-linear changing cross section, the three unknown values can be solved for with the help of software such as Maple and Microsoft Excel.

2.2 Weibull "Weakest Link" Model

2.2.1 Relationships of Weibull Weakest Link Model Describing Tensile Stress of GFRP

The Weibull weakest link model is incorporated to obtain the tensile strength from tensile rupture flexural modulus (Weil and Daniel 1964; Quinn and Quinn 2010). The probability of brittle material failure can be described as indicated in Equation [4].

$$P = 1 - \exp\left(-\int_V \left(\frac{\sigma-\sigma_u}{\sigma_o}\right)^m dV\right) \qquad [4]$$

where: V represents the volume of the specimen, and σ represents the applied stress, σ_u represents the zero strength stress where no failure occurs below this stress (which is usually assumed to be zero), σ_o is the normalizing factor (the scale parameter), and m is the Weibull modulus (shape parameter), which will be discussed in the next section.

If we consider that the probability of failure for both tension and flexure tests are to be the same, we can write:

$$\int_V \left(\frac{\sigma_t-\sigma_u}{\sigma_o}\right)^m dV_t = \int_V \left(\frac{\sigma_b-\sigma_u}{\sigma_o}\right)^m dV_b \qquad [5]$$

where σ_t refers to direct tensile stress and σ_b refers to flexural tensile stress (rupture modulus for flexure test for GFRP material). V_t and V_b represent the effective volumes experiencing tensile stress in a uniaxial direct tensile test and a flexural test, respectively.

After the completing the integration over the specimen for either tension or flexure Equation [5] is reduced to Equation [6]:

$$V_t \left(\frac{\sigma_t}{\sigma_o}\right)^m = V_b \left(\frac{\sigma_b}{\sigma_o}\right)^m \left(\frac{1}{2(m+1)^2}\right) \qquad [6]$$

Rearranging Equation [6] to solve for the ratio between the rupture modulus and the tensile strength gives Equation [7]. Equation [7] was derived in a general form to describe the ratio between two different stresses of materials that follow the Weibull weakest link model (Weil and Daniel 1964; Quinn 2003; Quinn and Quinn 2010).

$$\frac{\sigma_b}{\sigma_t} = \left(\frac{2V_t(m+1)^2}{V_b}\right)^{\frac{1}{m}} = = \left(\frac{V_{Et}}{V_{Eb}}\right)^{\frac{1}{m}} \qquad [7]$$

where: V_{Et} and V_{Eb} refer to the effective volumes experiencing tensile stress in a uniaxial direct tensile test and a flexural test, respectively. The tensile strength of GFRP bars can be found after obtaining the rupture modulus from flexural testing and calculating Weibull modulus and the effective volumes, (Eq. 7). The full derivation can be found in Lochan and Polak (2022)

2.2.2 Weibull Modulus

The Weibull modulus, also known as the shape parameter, is used to describe the distribution of GFRP material failure, which is linked to the flaws present in the material. This parameter's value can be obtained by using the Weibull strength distribution graph, where the natural logarithm of the rupture modulus is taken and plotted against the double natural logarithm of its respective probability in the list of samples, using Equation 8, where "n" represents the total number of specimens and "i" represents the rank of the specimen's strength relative to the others, in order from least to greatest.

$$P_f = \frac{i-0.5}{n} \qquad [8]$$

From these transformed points of data, a line of best fit can be plotted against the dataset, from which the Weibull modulus can be found by taking the slope of this line of best fit. It should be noted that this value becomes more accurate with having more sample data. Figure 2 shows one of the completed examples of the Weibull strength distribution graph for the tested M13 specimens.

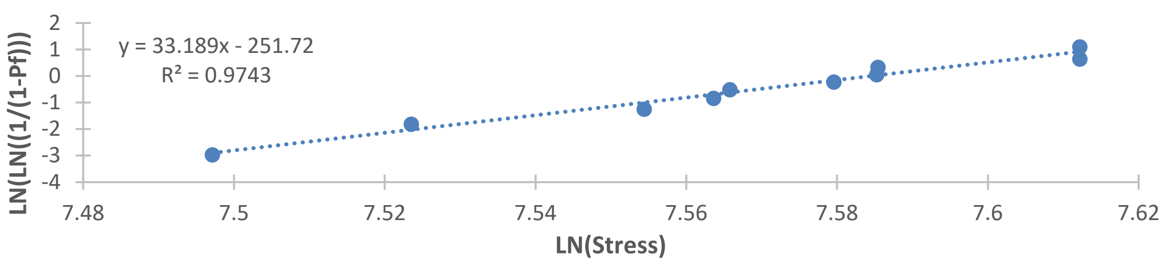

Figure 2: Weibull Strength Distribution Graph for M13 Specimens

2.2.3 Effective Volume Calculations

The calculation for the effective volume for tensile testing is simple, as it is the volume of the entire specimen, as noted in Equation [9].

$$V_{Et} = V_t = V = \pi R^2 L \qquad [9]$$

The calculation for effective volume for flexural testing requires consideration of the volume subjected to tension which can be defined by the bending moment diagram. The resulting equation is shown in Equation [10] (Lochan and Polak 2022).

$$V_{Eb} = \left(-\frac{2^{m+1}}{\left((h-c)L\right)^m}\right)\int_0^{\frac{L}{2}}\int_0^{-(h-c)}\int_{-\sqrt{R^2-\left(y-\left(R-(h-c)\right)\right)^2}}^{\sqrt{R^2-\left(y-\left(R-(h-c)\right)\right)^2}} y^m x^m \, dzdydx \qquad [10]$$

where: L is the length of the specimen, R is the original radius of the specimen, z-coordinate refers to the width of the cross section, y-coordinate refers to the height/depth of the cross-section, and the x-coordinate refers to the length of the cross-section. Due to the complexity of Equation [10], the mathematical software Maple was used to evaluate this integral after all the cross-sectional parameters were measured directly from the specimens. After solving for the effective volumes, Equation [7] can be used to determine the tensile strength of a GFRP bar.

3. TESTING PROCEDURE AND CALCULATIONS OF RUPTURE MODULUS

Using ASTM D4476-14 as a guideline for conducting the three-point bending tests, the GFRP bars were cut longitudinally via waterjet cutting to be prepared for testing. The testing apparatus and supports for the specimen that was used for the flexural tests, indicated in Figure 3.

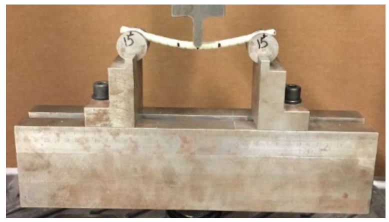

Figure 3: Flexure Testing Apparatus, Specimen Supports and Loading Nose for Three-Point Bending During a Three-Point Bending Test of a M13 Specimen

The cracking load is obtained from load displacement graphs as the maximum load where the relationship becomes nonlinear (Lochan and Polak 2022). The actual elastic moduli for the specimens were not tested in this research. Therefore, the values obtained by Arczewska et al. (2019) were used in the calculations. These results were obtained on very similar bars (M16). It is also worth noting that it is the ratio between tensile and compressive moduli that affects the predicted modulus of rupture. In this case the tensile modulus was taken as 63 GPa, compressive modulus as 53 GPa, and their ratio of 1.19. This ratio is consistent with recommendations from other researchers (Jones 1977, 1978; Savchenko 2005).

4 RESULTS OF TESTING AND DISCUSSION

The calculations done in this work are compared to the direct tensile testing results done in this research and to tensile strength tested by the manufacturers (Table 1). Note that the nominal tensile strength of the bars is 1000 MPa. Tensile testing was done only on M8, M13 and M15 bars. Manufacturer's specifications were available for M13, M15 and M20 bars. Larger bars could not be tested in direct tension due to laboratory constraints of having a testing frame capable of achieving high loads and testing are long bar lengths.

All reported values are within reasonable limits and exceed 1000 MPa, which is the required nominal (minimal) strength of these bars. Also, the correlated tensile capacities generally decrease as bar size increases which aligns with the size effect in brittle materials: where the larger the volume of the material, the more flaws are present, which lowers its capacity.

5 CONCLUSIONS

The paper presents experimental and theoretical research on using 3-point flexure testing of GFRP bars to determine their tensile strength. Direct tensile testing is difficult to conduct on GFRP bars and thus it is not routinely used for quality control, however the proposed methodology shows great promise for practical applications of determining tensile strength from flexural tests. The methodology uses the concept of bending of elastic bimodular half-circular cross sections and the Weibull's weakest link model to correlate bending and tensile results.

Table 1: Summary of Average Calculated Tensile Capacity Ratios, Correlated Tensile Capacities, and Tensile Capacities

Parameter	M8	M13	M15	M20	M25	M32
$\frac{\sigma_b}{\sigma_t} = \left(\frac{V_{Et}}{V_{Eb}}\right)^{\frac{1}{m}}$	1.5	1.5	1.6	1.5	1.4	1.5
$\boldsymbol{\sigma_t}$ **(MPa)**	1492	1253	1134	1201	1163	1077
$\boldsymbol{\sigma_{tt}}$ **(MPa)**	1344	1231	1209	-	-	-
% Diff.	10.4	1.8	6.4	-	-	-
$\boldsymbol{\sigma_{t.spec}}$ **(MPa)**	-	1487	1219	1278	-	-
% Diff.	-	17.1	7.3	6.3	-	-
σ_t correlated tensile strength σ_{tt} tensile strength from direct testing done in this research $\sigma_{t,spec}$ tensile strength from direct testing from manufactures specifications sheet.						

6 ACKNOWLEDGEMENTS

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